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ON THE INSTABILITY OF ELASTIC SHELLS AS THE MANIFESTATION OF INTERNAL RESONANCE*

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A large number of internal resonances, sensitivity to small imperfections and to a small external non-conservative action are characteristic for a number of elastic shells subjected to conservative forces. It is shown that, in combination, these three features result in dynamic instability of a system, that manifests itself in the existence of a solution of the explosive instability type when the deviation from the equilibrium state becomes infinitely large in a finite time. A simple method is proposed to calculate the ultimately allowable load by which one should be guided in designing structures containing thin shells. This load calculated by a linear model corresponds to the appearance of the first internal resonance in the system. The results are illustrated by well-known experimental facts.

Investigation of the stability of loaded elastic shells within the framework of a linearized model yields the so-called upper critical load T_1. A result of the solution of the stability problem in the large (non-linear static model) is the lower critical load T_2. For many objects (for instance, a cylindrical shell under uniform lateral compression) T_1 and T_2 are quite close and in good agreement with the limit load T_e achievable in experiment.

Meanwhile, there is a broad class of systems (among which, for instance, are a cylindrical shell compressed along the generatrix, a spherical shell under hydrostatic pressure) for which T_1 and T_2 differ substantially, where the load T_2 is quite sensitive to small corrections in the system model. A large spread in the values of the limit loads T_e is observed in tests. It is difficult to exclude the possibility of unexpected failures /1/ when designing structures whose elements are such shells.

Three characteristic features distinguish system of this class from shells for which the linearized model yields critical loads close to the experimental values: they are sensitive to small imperfections (initial deflection, inhomogeneity of the properties, etc.); the critical load depends strongly on the nature of the loading; internal resonances appear in a system starting at a certain value of an increasing load /2/.

These features explain why examination of the stability problems of such systems in the small or in a non-linear static formulation is not successful. The need for a dynamic approach to stability problems is also noted in experimental investigations of loaded shells /3/.

The absence of at least one of the factors mentioned results in the upper critical load in experiment. Thus, a load close to T_1 /4, 5/ is reached on cylindrical shells compressed in the axial direction because of careful shell fabrication and carrying out the experiment. Investigation of the stability in the small for such shells also yields a satisfactory result /6/.

We will turn to the simplest mechanical model containing the three features noted in order to show that in combination they can result in a dynamic instability that appears in an unbounded solution of the explosive instability type when the deviation from the equilibrium state becomes infinitely large in a finite time. We will then discuss the possibility of utilizing the results of the investigation of this model in shell analyses.

1. A system is considered that consists of three concentrated masses and massless stiff rods at whose hinge-connection sites an elastic restoring moment acts. The masses are fastened to non-linearly elastic springs, such that the expression for the elastic force includes linear and quadratic terms in the deviation. There is an initial deflection $\varphi_0 \ll 1$ (Fig.1). One end of the system is hinge-supported, while a force having a constant component T and a non-conservative part ΔT is applied to the other in a vertical direction.

Using the Lagrange method, we obtain the following system equations of motion:

$$3\varphi_1^{\cdot\cdot} + 2\varphi_2^{\cdot\cdot} + \varphi_3^{\cdot\cdot} + 2\varphi_1 - \varphi_2 + \varkappa(3\varphi_1 + 2\varphi_2 + \varphi_3) - \quad (1.1)$$

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$$\tau\varphi_1 = \varphi_2^{\cdot\cdot}(\varphi_1 - \varphi_2)^2 + {}^1/_2\varphi_3^{\cdot\cdot}(\varphi_1 - \varphi_3 - \varphi_0)^2 -$$
$$2\varphi_2^{\cdot 2}(\varphi_1 - \varphi_2) - \varphi_3^{\cdot 2}(\varphi_1 - \varphi_3 - \varphi_0) - \varkappa_1(3\varphi_1^2 + 2\varphi_2^2 +$$
$$4\varphi_1\varphi_2 + 2\varphi_2\varphi_3 + 2\varphi_1\varphi_3) - {}^1/_6\,\tau\varphi_1^3 + Q_1$$
$$2\varphi_1^{\cdot\cdot} + 2\varphi_2^{\cdot\cdot} + \varphi_3^{\cdot\cdot} + 2\varphi_2 - \varphi_1 - \varphi_3 + \varkappa(2\varphi_1 + 2\varphi_2 + \varphi_3) -$$
$$\tau\varphi_2 = \varphi_1^{\cdot\cdot}(\varphi_1 - \varphi_2)^2 + {}^1/_2\varphi_3^{\cdot\cdot}(\varphi_2 - \varphi_3 - \varphi_0)^2 +$$
$$2\varphi_1^{\cdot 2}(\varphi_1 - \varphi_2) - \varphi_3^{\cdot 2}(\varphi_2 - \varphi_3 - \varphi_0) -$$
$$\varkappa_1(2\varphi_1^2 + 2\varphi_2^2 + \varphi_3^2 + 4\varphi_1\varphi_2 + 2\varphi_1\varphi_3 + 2\varphi_2\varphi_3) - {}^1/_6\,\tau\varphi_2^3 + Q_2$$
$$\varphi_1^{\cdot\cdot} + \varphi_2^{\cdot\cdot} + \varphi_3^{\cdot\cdot} + \varphi_3 - \varphi_2 + \varkappa(\varphi_1 + \varphi_2 + \varphi_3) -$$
$$\tau(\varphi_3 + \varphi_0) = {}^1/_2\varphi_1^{\cdot\cdot}(\varphi_1 - \varphi_3 - \varphi_0)^2 + {}^1/_2\varphi_2^{\cdot\cdot}(\varphi_2 - \varphi_3 -$$
$$\varphi_0)^2 + \varphi_1^{\cdot 2}(\varphi_1 - \varphi_3 - \varphi_0) + \varphi_2^{\cdot 2}(\varphi_2 - \varphi_3 - \varphi_0) -$$
$$\varkappa_1(\varphi_1 + \varphi_2 + \varphi_3)^2 - {}^1/_6\,\tau(\varphi_3^3 + 3\varphi_3\varphi_0^2 + 3\varphi_3^2\varphi_0) + Q_3$$

Here terms not higher than cubic in the angles φ_i are present since we will later limit ourselves to a quadratic non-linearity in going over to the equations in deviations. The quantity $t_* = (ml^2c^{-1})^{1/2}$ is selected as the time scale and the notation $\varkappa = kl^2c^{-1}$, $\varkappa_1 = k_1l^3c^{-1}$, $\tau = Tlc^{-1}$, is introduced where c is the spring elasticity factor at sites of hinge connection of the rods, and the parameters k and k_1 characterize the springs connecting the mass to a fixed support.

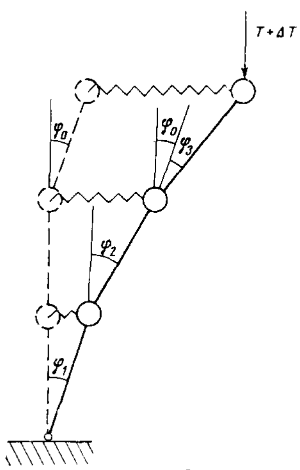

Fig.1

The generalized forces Q_i have a form similar to the expressions for the conservative load components on the left-hand sides of (1.1),the only difference being that τ is replaced by $\Delta\tau$:

$$Q_{1,2} = \Delta\tau\varphi_{1,2}, \quad Q_3 = \Delta\tau(\varphi_3 + \varphi_0)$$

We consider the non-conservative force $\Delta\tau = \Delta Tlc^{-1}$ related to the following way to the vertical velocity component of the upper mass

$$\Delta\tau^{\cdot} + \alpha\Delta\tau = -\beta\Delta x_3^{\cdot}\ (\alpha, \beta \geqslant 0)$$

Apart from quadratic terms in the deflection of the rods from the vertical, the change in the upper mass coordinate is given by the relationship

$$\Delta x_3 = {}^1/_2(\varphi_1^2 + \varphi_2^2 + \varphi_3^2) + \varphi_3\varphi_0$$

The loading unit can be a gas-filled cylinder with two pistons, say, one of which is connected to the mass m_3. A definite gap between the pistons corresponds to the load T. By virtue of the inertia of the unit, a change in x_3 is followed by the second piston with a lag. A small non-conservative load thus occurs.

The construction of the loading unit is not so important in this analysis. It is essential that a small non-conservative force $(\sim\varphi_j^2)$ that is ordinarily missing in the linearized or non-linear static models being studied, acts on the system together with the conservative load T.

Because of the initial deflection under the load $\tau \neq 0$ the system has an equilibrium state that differs from the original (shown in Fig.1). Linearizing (1.1) near the equilibrium state, the natural frequencies and their corresponding system vibration modes can be determined.

The case when internal resonance is observed in the system, i.e., the frequencies are connected by the relationship $\omega_1 + \omega_2 = \omega_3$ is of interest. A further analysis is performed for $\varkappa = 2.5$, $\tau = 2.08$, to which the frequencies $\omega_1 = 0.92$, $\omega_2 = 1.52$, $\omega_3 = 2.44$ correspond. We note that all the quantities ω_j are real for $\varkappa = 2.5$ under loads τ from the interval /0, 2. 59/.

We will change to normal coordinates εq_j, $\varepsilon \ll 1$. (Since all the φ_j are assumed small, the parameter ε is extracted). Eqs.(1.1) are transformed to the form

$$q_j^{\cdot\cdot} + \omega_j^2 q_j = \varepsilon f_j(q_k), \quad j = 1, 2, 3, \tag{1.2}$$

where f_j are quadratic functions of q_k $(k = 1, 2, 3)$ and their conjugates.

We limit ourselves to just the "resonance" terms in the expressions for f_j

$$f_1 = i\,33.55D_1\varphi_0^2q_1 + q_2^*q_3\,[0.21\,\varkappa_1 + 14.35\varphi_0 +$$
$$i\varphi_0\,(1.67\,D_1 - 1.22\,D_2^* + 3.49\,D_3)]$$
$$f_2 = i\,2.98D_2\varphi_0^2q_2 + q_1^*q_3\,[0.2\varkappa_1 + 13.32\,\varphi_0 +$$
$$i\varphi_0\,(-1.56D_1^* + 1.11D_2 + 3.18D_3)]$$
$$f_3 = i20.08D_3\varphi_0^2q_3 + q_1q_2[0.71\varkappa_1 + 47.98\varphi_0 +$$
$$i\varphi_0\,(5.56D_1 + 4.08D_2 + 11.66D_3)]$$
$$D_j = -\beta\,(\alpha - i\omega_j)(\alpha^2 + \omega_j^2)^{-1}$$

We seek the solution of system (1.2) in the form

$$q_j = A_j\,(\eta = \varepsilon t)\exp\,(i\omega_j t) + \varepsilon w_j\,(t)$$

After substituting q_j into the equations and equating terms of identical order in ε we obtain

$$w_j^{\cdot\cdot} + \omega_j^2 w_j = -2i\omega_j(dA_j/d\eta)\exp\,(i\omega_j t) + f_j\,(A_k \exp\,(i\omega_k t))$$

The error w_j will not increase if the right-hand side of this relationship is orthogonal to the eigenfunctions of the problem for $\varepsilon = 0$. Then the equations for the amplitudes will be the following

$$2i\omega_j(dA_j/d\eta) = \langle f_j\,(A_k \exp\,(i\omega_k t))\exp\,(-i\omega_j t)\rangle$$

where $\langle\ldots\rangle$ denotes the average in the time $t_0 \gg T_k$ $(T_k = 2\pi/\omega_k)$.

After taking the average and going over to the real amplitudes and phases a_l, δ_l by using the change of variables

$$A_l = a_l\,(|\sigma_j||\sigma_k|)^{-1/2}\exp\,(i\delta_l);\ j \neq k,\ j,\ k \neq l$$

we obtain the equations

$$\begin{aligned} &da_1/d\eta = a_2a_3\cos\,(\Phi + \theta_1) + \chi_1 a_1\,(1, 2, 3) \\ &d\Phi/d\eta = -\frac{a_2a_3}{a_1}\sin\,(\Phi + \theta_1) - \frac{a_1a_3}{a_2}\sin\,(\Phi + \theta_2) - \\ &\frac{a_1a_2}{a_3}\sin\,(\Phi + \theta_3);\quad \Phi = \delta_3 - \delta_1 - \delta_2 \\ &\cos\theta_j = \frac{\mathrm{Re}\,\sigma_j}{|\sigma_j|},\quad \sin\theta_{1,2} = \frac{\mathrm{Im}\,\sigma_{1,2}}{|\sigma_{1,2}|},\quad \sin\theta_3 = \frac{\mathrm{Im}\,\sigma_3}{|\sigma_3|} \end{aligned} \tag{1.3}$$

$$\begin{aligned} &\sigma_j = -\alpha\beta\varphi_0 m_j - i[\xi_j + \varphi_0\,(\zeta_j - \beta n_j)],\ j = 1,\ 2,\ 3 \\ &\xi_1 = 0.12,\ \xi_2 = 0.07,\ \xi_3 = 0.15 \\ &\zeta_1 = 7.78,\ \zeta_2 = 4.4,\ \zeta_3 = 9.84 \\ &n_1 = 0.83d_1 + 1.01d_2 + 4.61d_3,\ n_2 = 0.47d_1 + 0.56d_2 + 2.56d_3 \\ &n_3 = 1.05d_1 + 1.27d_2 + 5.83d_3,\ m_1 = 0.9d_1 - 0.66d_2 + 1.89d_3 \\ &m_2 = -0.51d_1 + 0.37d_2 + 1.05d_3,\quad m_3 = 1.14d_1 + 0.84d_2 + 2.39d_3 \\ &\chi_j = -\nu_j\alpha\beta\varphi_0^2 d_j,\ \nu_1 = 18.19,\ \nu_2 = 0.98,\ \nu_3 = 4.12 \\ &d_j = (\alpha^2 + \omega_j^2)^{-1}. \end{aligned}$$

We note that when changing to real amplitudes and phases, there are still two equations in addition to (1.3) by which the quantity δ_l can be determined after (1.3) has been solved.

When there is no initial deflection $(\varphi_0 = 0,\ \theta_{1,2} = \pi/2,\ \theta_3 = 3\pi/2)$ the system is conservative. Only energy exchange between the modes is possible in it. An analogous situation holds for $\varphi_0 \neq 0$, $\alpha = 0$ or (and) $\beta = 0$.

The initial deflection and the small $(\sim q_j^2)$ non-conservative force from the loading unit $(\alpha \neq 0,\ \beta \neq 0)$ jointly produce conditions when an explosive instability becomes possible in the system: the amplitudes of all the resonantly associated modes grow, where the solution becomes unbounded in the finite time t_∞ similar to $(t_\infty - t)^{-1}$ /8/.

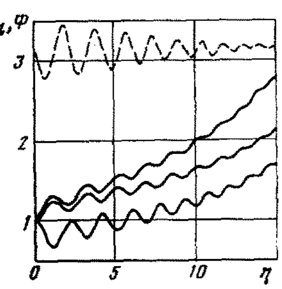

Fig.2

Without taking account of terms in (1.3) that contain χ_j, the necessary condition for the existence of a solution of the system of an explosive instability type is the arrangement of all θ_j in one half-plane. Taking account of these terms (they are responsible for energy dissipation in the model under consideration) as a function of the magnitudes of the parameters χ_j results in either an increase in the time t_∞ or in the impossibility of an explosive instability /8/.

The time dependences of the amplitudes a_l (the solid lines) and the phases Φ (the dashed line) obtained by numerical integration of system (1.3) for $\alpha = 1$, $\beta = 1$, $\varphi_0 = 0.01$, to which

$$\theta_1 = 3\pi/2 - 3.06\times10^{-2},\quad \theta_2 = 3\pi/2 + 1.36\times10^{-3},\quad \theta_3 = \pi/2 + 5.36\times10^{-2}$$

$$\chi_1 = -9.82\times10^{-4},\ \chi_2 = -0.3\times10^{-4},\ \chi_3 = -0.59\times10^{-4}$$

correspond and for the initial conditions $a_1\,(0) = a_2\,(0) = a_3\,(0) = 1$, $\Phi\,(0) = \pi$ are presented in Fig.2. All the θ_j are in one half-plane $(\theta_1 - \theta_3 < \pi)$. In the case under consideration this condition is not only necessary but also sufficient for the system (1.3) to have a solution of the explosive instability type.

Let us estimate the time t_∞. Since one of the amplitudes grows fivefold in the time $\sim 20\varepsilon t$, for $\varepsilon = 10^{-2}$ the time t_∞ corresponds to $\sim 5\times10^2$ vibration cycles at the lower frequency.

Calculations performed without taking account of energy dissipation show that the nature of the amplitude growth for the given parameters α, β, φ_0 does not vary so noticeably.

Hence, we have shown that an explosive instability is possible in the system. It is due to an internal resonance, an initial deflection, and a small non-conservative force.

It can be seen that for all $\tau \in [0, 2.59]$ the system under consideration is stable in the small with the exception of the resonance domain, a small neighbourhood $\tau = 2.08$.

2. We will use the example of a cylindrical shell compressed in the axial direction to discuss the possibility of utilizing the results of studying a three-mass model in the solution of problems of the stability of loaded shells.

We direct the x and y coordinate lines, respectively, along the generatrix and arcs of the cylindrical shell. Let $w(x, y, t)$ denote the shell deflection and let us assume that there is no initial deflection.

The equations of shell vibrations have the form /9/

$$\rho \frac{\partial^2 w}{\partial t^2} + \frac{D}{h} \nabla^4 w + T \frac{\partial^2 w}{\partial x^2} - \frac{1}{R} \frac{\partial^2 \Phi}{\partial x^2} + \tag{2.1}$$

$$\frac{\rho}{E}(1+\sigma)\left(\frac{1}{R} + \nabla^2 w\right)\frac{\partial^2 \Phi}{\partial t^2} - L(w, \Phi) = 0$$

$$\frac{1}{E}\left[\nabla^2 - \frac{\rho}{E}(1-\sigma^2)\frac{\partial^2}{\partial t^2}\right]\left[\nabla^2 - \frac{2\rho}{E}(1+\sigma)\frac{\partial^2}{\partial t^2}\right]\Phi +$$

$$\frac{1}{R}\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} L(w, w) = 0$$

$$D = Eh^3 \frac{1}{12(1-\sigma^2)}, \quad \nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$$

$$L(w, w) = 2\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]$$

$$L(w, \Phi) = \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 w}{\partial y^2}\frac{\partial^2 \Phi}{\partial x^2} - 2\frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 \Phi}{\partial x \partial y}$$

where Φ is the dynamic stress function in the middle surface, D is the cylindrical stiffness, R, l, h are, respectively, the radius, length, and thickness of the shell, ρ is the density, E is the modulus of elasticity, and σ is Poisson's ratio.

We are interested primarily in the normal vibrations modes to which significantly lower frequencies correspond than to the tangential modes. Consequently, in the problem under consideration the law of dynamic stress function variation with time is determined by the deflection, and the influence of the motion in the middle surface on Φ should be neglected, i.e., the terms containing the time derivatives in the second equation of (2.1) should henceforth not be taken into account.

Assuming that simple supprt conditions are satisfied at the shell endfaces, we approximate the deflection by the expression

$$w(x, y, t) = \sum_{m, n} A_{mn}(t) \sin m\xi \sin n\zeta$$

$$(\xi = x\pi/l, \ \zeta = y/R; \ 0 \leqslant x \leqslant l, \ 0 \leqslant y \leqslant 2\pi R)$$

As the load T increases, starting with a certain value T_* internal resonances are possible in the system, which is associated with the singularity in the behaviour of the spectrum $\omega_j(T)$ in problems of this class: the frequencies corresponding to modes with a very large number of inflections undergo the greatest variation. Even the approximate estimate where only axisymmetric modes were taken in the analysis yields 10^2 - 10^3 resonance points /2/ in the interval $[T_*, T_1]$.

For a certain load T let the internal resonance conditions be satisfied, which for a system with distributed parameters have the form

$$m_1 + m_2 = m_3, \ n_1 + n_2 = n_3, \ \omega_1 + \omega_2 = \omega_3$$

After substituting the expression for $w(x, y, t)$ from the second Eq. (2.1), we find the stress function $\Phi(x, y, t)$ and substitute it into the first equation of the system. Then using the Galerkin method, we find equations for the amplitudes of the modes in resonance, apart from quadratic terms

$$\ddot{A}_{m_1n_1} + \omega_1^2 A_{m_1n_1} = f_{11}(c_2b_3 - c_3b_2)^2 A_{m_2n_2}A_{m_3n_3} + \tag{2.2}$$

$$f_{21}\left(\frac{a_2b_3^2}{a_3^2} A_{m_2n_2}\ddot{A}_{m_3n_3} + \frac{a_3b_2^2}{a_2^2}\ddot{A}_{m_2n_2}A_{m_3n_3}\right)$$

$$\ddot{A}_{m_2n_2} + \omega_2^2 A_{m_2n_2} = f_{12}(c_1b_3 - c_3b_1)^2 A_{m_1n_1}A_{m_3n_3} +$$

$$f_{22}\left(\frac{a_1b_3^2}{a_3^2} A_{m_1n_1}\ddot{A}_{m_3n_3} + \frac{a_3b_1^2}{a_1^2}\ddot{A}_{m_1n_1}A_{m_3n_3}\right)$$

$$\ddot{A}_{m_3n_3} + \omega_3^2 A_{m_3n_3} = f_{13}(c_2b_1 - c_1b_2)^2 A_{m_1n_1}A_{m_2n_2} +$$

$$f_{23}\left(\frac{a_1b_2^2}{a_2^2} A_{m_1n_1}\ddot{A}_{m_2n_2} + \frac{a_2b_1^2}{a_1^2}\ddot{A}_{m_1n_1}A_{m_2n_2}\right)$$

$$\omega_j^2=\frac{D}{\rho h}\left(a_j^2+\frac{Eh}{DR^2}\frac{b_j^4}{a_j^2}-\frac{Th}{D}b_j^2\right)d_j$$

$$f_{1k}=\frac{E}{\rho h}d_k\sum_{j=1}^{3}\left(\frac{b_j}{a_j}\right)^2,\quad f_{2k}=\frac{\rho(1+\sigma)}{R}d_k$$

$$a_j=b_j^2+c_j^2,\quad b_j=\frac{\pi m_j}{l},\quad c_j=\frac{n_j}{R},\quad d_j=\left[1+\frac{1+\sigma}{R^2}\frac{b_j^2}{a_j^2}\right]^{-1}$$

We note that the upper critical load is calculated from the expression for ω_j^2. Setting $\omega_j^2=0$, and minimizing T by $z_j=(a_j/b_j)^2$, we find $T_1=EhR^{-1}[3(1-\sigma^2)]^{-1/2}$.

After taking into account the initial deflection and the small non-conservativeness of the loading unit, problem (2.2) reduces to system (1.3) considered earlier. Therefore, the results of investigating the three-mass model have a direct relation to the stability problems of loaded shells.

The main deduction is that the problems of the dynamics of elastic shells loaded by conservative forces allow a solution of explosive instability type when small imperfections and a small external non-conservative action are taken into account.

When examining specific structures containing thin shells, it is not realistic to count on sufficient information about their imperfections (particularly the initial deflection) and the nature of the loading; consequently, one should consider the load T_* at which the possibility of an explosive instability first occurs, as the greatest allowable load. This load corresponds to the appearance of the first internal resonance in the system and is determined from examination of a substantially simpler linear conservative model than the original.

In practical computations the load for which the branches of $\omega_j(T)$ first intersect should be taken as T_*. In the neighbourhood of this value of the load there are slightly differing frequencies corresponding to adjacent m, n which together with the small low frequency satisfy the resonance conditions.

In /2/ where only axisymmetric modes were taken into account $(c_j=0)$, the loads T_* was calculated as the maximum value of the load when the condition $d\omega_j^2/db_j^2\geqslant 0$ is satisfied for all b_j^2.

In the general case $(c_j\neq 0)$ the critical load T_* agrees with that found for the axisymmetric modification. This can be seen if a new variable α is itroduced in place of c_j^2 such that $c_j^2=\alpha b_j^2$. Then the equivalence of the case $\alpha\neq 0$ to the simultaneous increase of the shell thickness and radius by a factor of $1+\alpha$ follows from the expression for ω_j^2 Since the ratio of these quantities is in the expression for T, we arrive at the result of the axisymmetric problem.

The ratio of T_* to the upper critical load equals

$$\frac{T_*}{T_1}=\frac{3}{2}\left[\frac{1+\sigma}{12(1-\sigma)}\right]^{1/4}\left(\frac{h}{R}\right)^{1/2}$$

We note that it differs somewhat from that presented in /2/ where a mistake is made in the expression for ω_j^2.

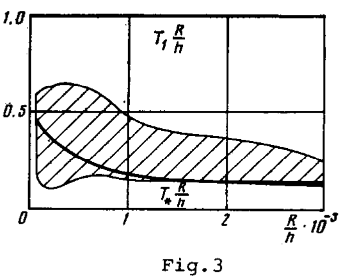

Fig.3

We show in Fig.3 are T_1, T_* for $\sigma=0.3$ (usually used in computations) and the domain of experimental critical loads (shaded). The load T_* agrees with the lower boundary of this domain in a broad range of R/h. The spread in the experimental data is explained by the fact that the presence of internal resonances in the system is just the necessary condition for explosive instability. The limit load for a specific system is also governed by its small imperfections, the nature of the loading and the initial conditions. For large shell thickness the lower boundary T_e does not coincide with the calculated load T_* since additional terms must be introduced in the system (2.1) for these h and therefore, the expression for ω_j^2 by which T_* is calculated changes.

In addition to the longitudinally compressed cylindrical shell, there is considerable experimental material in the literature on spherical shells under hydrostatic pressure (/9/ for instance). An analytic expression can also be obtained for the frequencies for a spherical shell and therefore the ratio between the load T_* corresponding to the appearance of internal resonance in the system and the upper critical load, having the form

$$\frac{T_*}{T_1}=\frac{3}{2}\left[\frac{(1+3\sigma)^2}{12(1-\sigma^2)}\right]^{1/4}\left(\frac{h}{R}\right)^{1/2}$$

can be calculated.

Comparison of the experimental data $T_e(R/h)$ with the calculated load T_* results in the deduction that T_* is close to the lower boundary of the T_e domain (as in the case of the longitudinally compressed cylindrical shell).

In conclusion, we note that the approach proposed in this paper to problems of the stability of loaded shells supplements the static investigations of shells (/4, 9/, say) and enables us to clarify the behaviour of real shells in a number of extraordinary cases. Indeed, there is still one stable equilibrium state in addition to the original in the $(T_2, T_1]$ load range for systems in this class. Finite shell deflection corresponds to it. The transition to this equilibrium state of a shell is completed by a jump for $T = T_1$ (in the idealized model). From the viewpoint of steady representations for $T < T_1$ small deviations do not take the shell out of the domain of attraction of the original equilibrium state. The presence of internal resonance, small non-conservative forces, and imperfections of the shell can result in rapid growth of these deflections, after which the jump follows.

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